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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

CALCULUS.

308. Proposed by C. N. SCHMALL, New York City.

Prove, by calculus, that of all isoperimetric triangles, the equilateral has the greatest area.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Letting s represent the half sum of the three sides x, y, z , we have for the square of the area, $s(s-x)(s-y)(s-z)$; therefore $m = (s-x)(s-y)(s-z) = (s-x)(s-y)(x+y-s) = xy(x+y) - (x^2 + 3xy + y^2)s + 2(x+y)s^2 - s^3$ is to be a maximum. By differentiation

$$\frac{\partial m}{\partial x} = y(2x+y) - (2x+3y)s + 2s^2,$$

$$\frac{\partial m}{\partial y} = x(2y+x) - (2y+3x)s + 2s^2,$$

$$\text{From } \frac{\partial m}{\partial x} = 0, \quad \frac{\partial m}{\partial y} = 0,$$

by subtracting one from the other, we get $(x-y)[s-(x+y)] = 0$; whence $x-y=0$, the other root $x+y=s$ not being available. Hence $y=0$.

Substituting in either one, we have $3x^2 - 5xs + 2s^2 = 0$; whence $x=s$, $x=\frac{2}{3}s$, the former of which is unavailable.

$$\therefore x = \frac{2}{3}s, \quad y = \frac{2}{3}s, \quad \text{and } z = \frac{2}{3}s.$$

II. Solution by the PROPOSER.

Let the given perimeter be $2s$; let the sides be x, y, z , and u the area. Then we have $z = 2s - x - y$. Now

$$u = \sqrt{s(s-x)(s-y)(x+y-s)} \dots (1),$$

which is to be rendered a maximum. Taking logarithms, we have

$$2 \log u = \log s + \log(s-x) + \log(s-y) + \log(x+y-s).$$

$$\therefore \frac{2}{u} \frac{\partial u}{\partial x} = -\frac{1}{s-x} + \frac{1}{x+y-s}. \quad \therefore \frac{\partial u}{\partial x} = \frac{u}{2} \frac{2s-2x-y}{(s-x)(x+y-s)} \dots (2),$$

$$\text{and } \frac{\partial u}{\partial y} = \frac{u}{2} \frac{2s-2y-x}{(s-y)(x+y-s)} \dots (3).$$

Equating (2) and (3) to 0, we have $2s-2x-y=0$, $2s-2y-x=0$, whence $x=\frac{2}{3}s$, $y=\frac{2}{3}s$, $z=2s-x-y=\frac{2}{3}s$.

Hence the triangle is equilateral; and there is evidently a maximum.

309. Proposed by S. G. BARTON, Ph. D., Clarkson School of Technology.

In practical problems involving maxima and minima, it is really the greatest and least values of the function which are desired. Show why we can assume that the maximum is the greatest value and the minimum the least value under the conditions.

No solution of this problem has been received.

310. Proposed by C. N. SCHMALL, New York City.

Evaluate $\int_0^\pi \frac{dx}{1-2a\cos x+a^2}$. Edwards' *Integral Calculus for Beginners*, page 131, ex. 9, (iii). The answer given is $\frac{\pi}{1-a^2}$. Is this a complete answer to the question?

I. Solution by H. PRIME, Boston, Mass.

$$\begin{aligned} \int_0^\pi \frac{dx}{1+a^2-2a\cos x} &= \left[\frac{2}{\pm(1-a^2)} \tan^{-1} \left(\pm \frac{1+a}{1-a} \tan \frac{x}{2} \right) \right]_0^\pi \\ &= \frac{(2n+1)\pi}{\pm(1-a^2)} - \frac{2n\pi}{\pm(1-a^2)} = \frac{\pi}{\pm(1-a^2)}. \end{aligned}$$

It should be noted that if $a=\pm 1$, the function $1/(1+a^2-2a\cos x)$ becomes infinite for $x=0$ or $x=\pi$. Hence, in this case, the integral has no meaning for the required limits, as they do not exist. The result also shows this for $\pi \div (1-a^2) = \infty$, when $a=\pm 1$.

Also solved by J. Scheffer, Francis Rust, A. M. Harding, and V. M. Spunar.

311. Proposed by WILMER THOMPSON, Senior, Drury College.

Solve the differential equation,

$$\left(\frac{dy}{dx} \right)^3 + x^3 = ax \left(\frac{dy}{dx} \right).$$

[From Forsythe's *Differential Equations*, p. 47.]